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INFORMATION PROPAGATION MODEL ON MULTILAYER SCALE-FREE NETWORKS

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ABSTRACT

People usually use multiple social networks simultaneously, and can share the information they learned from one social network to another. In this paper, we study the information spreading on multilayer networks and propose a model that the acceptance of information is not only determined by the information itself (fundamental transmissibility), but also influenced by the state of the individuals. In particular, layer-switching cost is taken into consideration in our model. The numerical results indicate that multilayer networks have the lower threshold and the larger propagation size than the single network. Furthermore, we get the threshold equation from the simulation results and prove that the structure of source layer plays a dominant role in the multilayer system.

KEYWORDS: Information Propagation, Multilayer Networks, Scale-Free, Threshold

INTRODUCTION

Many real world systems can be modeled as networks, i.e., sets of interconnected entities. A typical example is represented by online social networks, where information can move from one user to the other through, e.g., friendship or following connections. In recent years, the information diffusion process of complex networks has been an extensively discussed topic [1-4].

Although spreading processes of networks had been thoroughly studied during the last decade, real spreading phenomena are seldom constrained into a single network. In online social networks, individuals usually use different social platforms at the same time, and can share the information they have learned from one platform to another. For example, users of online social networks are often faced with a variety of options to choose from, e.g., Facebook, Twitter, in which they learn and spread information. In addition to studies on single-layer networks, increasing attention has been paid to epidemic and information diffusion on multilayer networks [5-8]. Buono*et al*. present a study of epidemic diffusion process of partially overlapped multiplex networks, in which nodes belonging to both layers help to reduce the epidemic threshold of the multiplex system[9]. Saumell-Mendiola *et al*. propose a model of epidemic spreading on top of two interconnected networks, and suggest that global epidemic may arise even the disease is incapable to propagate in either network separately[10]. Also, Yağan*et al*. found that overlaying social-physical networks can speed up information diffusion [11]. The study of epidemic and information diffusion on multilayer networks shows richer phenomenology and additional interdependencies, compared to the spreading process that takes place on a single network.

Relevant studies on complex networks are occasionally conducted analogously to the epidemic spreading process [12-15]. Epidemic models describing distinct disease spreading mechanisms are proposed, the most extensively studied ones including SI[16], SIS[17], SIR[18-20] models. In the SIR model, each node of the network has one of three possible states: susceptible (S), infected (I), and recovered (R). The SIR diffusion process of a single-layer network is equivalent to bond percolation with bond occupation probability as the transmissibility[14]. Suppose that the average infection rate between two individuals is *r*, and the infected one remains the infective state for a time *τ*, then the probability that the disease will not be propagated from the infected individual to the other is $1-T=\lim_{\delta t\to 0}(1-r\delta t)^{\tau/\delta t}=e^{-r\tau}$, so the probability of transmission is $T=1-e^{-r\tau}$. We call *T* the "transmissibility", which denotes an infected individual infects its neighbors before recovery.

As in the SIR model, an individual cannot be reinfected, the disease spreads through branches of infection that have a tree-like structure, and thus can be described using a generating function formalism[21, 22] that holds in the thermodynamic limit. In the generating function framework, the relevant magnitude that provides information about the process is the probability *f* that a branch of infection can extend throughout the network[23].When a branch of infection reaches a node of connectivity *k* across one of its links, the branch can only expand through its remaining $k-1$ connections. Thus the probability that a node of connectivity *k* belongs to a branch of infection is proportional to $k[1-(1-Tf)^{k-1}]$, since the probability to reach a node through a link is proportional to its connectivity. Thus *f* verifies the self-consistent equation $f=1-G_1(1-Tf)$ in isolated networks, where $G_1(x)=\sum_k kP(x)/\langle k\rangle x^{k-1}$ is the generating function of the underlying branching process[22], *P(x)* is the degree distribution, and $\langle k\rangle$ is the average degree of the network. In the steady state of the epidemics, the branches of infection from a single cluster of recovered individuals made up of nodes that were infected by some of its connections. Thus the fraction of nodes in the cluster of infection of an isolated network is given by $R=1-G_0(1-Tf)$, where $G_0(x)=\sum_k P(k)x^k$ is the generating function of the degree distribution. Within this formalism, the self-consistent equation has a nontrivial solution above the critical transmissibility $T_c=1/(\kappa-1)$, where $\kappa=\langle k^2\rangle/\langle k\rangle$ is the branching factor and $\langle k^2\rangle$ is the second moment of *P(k)*.

METHOD AND MODEL

However, the derivations above are based on a premise that the transmissibility is constant. In real online social networks, the propagation of information usually proceeds with complicated ways determined by many human behavioral and psychological factors. In this paper, we propose a model with the adjusted transmissibility to study how the connections between layers affect the spreading of information. We start by defining the specific setup of our model. The multilayer system consists of a source layer and a non-source layer with each layer representing an online social network. The two layers of the same size *N* represent two different groups of user accounts respectively. The layers have different connectivities with a random degree for each node, and we incorporate no degree correlations between the layers. In the multilayer system, the connections between two layers in Figure 1(a) show that a part of individuals own an account on both networks, allowing them (connection nodes) to participate in the information diffusion process on both layers. Here, the fraction of connection nodes in each layer called connection rate, denote by *θ*. *θ* can be expressed by

$$\theta=\frac{N_c}{N}, \quad (1)$$

Where N_c is the number of connections, and *N*is the total number of nodes in each layer.

Figure 1(a) shows an example of the information diffusion process takes place on multilayer networks. All individuals in the system are ignorant of the information at the initial stage. The diffusion process begins by randomly choosing an individual in source layer as the source of information. Then the information will spread from this node to its neighbors with the transmissibility T and propagate on the network. When a connection node accepts the information and becomes infected, its counterpart in another layer will be activated because they present the same individual. Then the information will spread from the counterpart to its neighbors under the effect of layer-switching cost[24]. Here, the layer-switching cost ratio α is introduced to describe the effect of layer-switching cost. Then the transmissibility can be expressed by

$$T_s = (1-\alpha)\times T\ , \qquad \textbf{(2)}$$

Where T is the fundamental transmissibility, and α is necessarily always in the range $0 \le \alpha \le 1$. The presence of the layer-switching cost, describing the overburden or surcharge for transmissions that proceed by crossing different layers compared to those proceeding as confined to the same layer. For example, when one received new information on an online social medium, say Twitter, he would more likely spread it again through Twitter as it is the handiest, than would do it over other online media, such as e-mail, let alone to an offline social network, as it would require additional effort and accompany spatiotemporal delay in switching the medium. In Figure 1(a), the information propagates between two layers under the effect of the layer-switching cost. The nodes of different colors indicate that the individuals receive information on different layers.

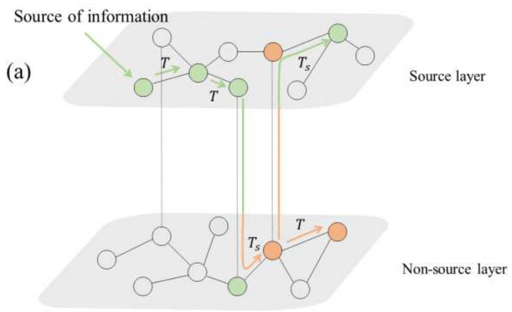

Figure 1(a): Schematic Illustration of the Information Diffusion Process with Layer-Switching Cost on the Multilayer Networks

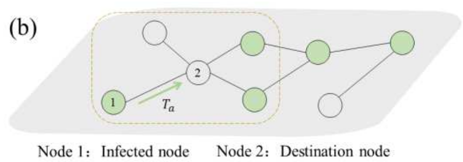

Figure 1(b): Schematic Illustration of the Adjusted Transmissibility in the Information Diffusion Process

In real social networks, people usually refer to or follow the views of the surrounding during the process of the information-spreading. To generate a more realistic scenario, we propose a framework that the acceptance of a node is not only determined by the fundamental transmissibility T, but also influenced by the state of its neighbors. Figure 1(b) shows an example of the information diffusion process, the green nodes represent the infected individuals, and then we use T_a instead of the fundamental transmissibility T of the diffusion process

$$T_a = T \times T^{\frac{1}{2} - \frac{N_i}{N_a}}, \quad (3)$$

Where N_i is the number of infected neighbors and N_a is the number of all neighbors of the destination node respectively. This means that the destination node will be more likely to accept the information when more than half of its neighbors are already infected, and the probability increases from the proportion of the infected neighbors. On the contrary, the information will be more difficult to be accepted.

SIMULATIONS AND ANALYSIS

To further assist understanding the proposed model, we give a set of simulations based on the scale-free networks. Scale-free networks can excellently model a great number of real world systems where the degree distribution follows a power-law rule as $P_k \propto k^{-\gamma}$ with γ being the scaling exponent. The parameters used to generate the Barabasi-Albert (BA) scale-free network[25] are: $N = 10^4$, $m_0 = m = 3$ and $\langle k \rangle = 6$. All the simulations contain 10^5 realizations in this paper.

In Figure 2, results of the stationary fraction of recovered individuals on a single network are presented. The simulation results show that our model with the adjusted transmissibility has a higher threshold. The value of threshold increases from $T_c = 0.105$ to $T_c = 0.188$. This indicates the consideration about the state of individuals will suppress the spread of information.

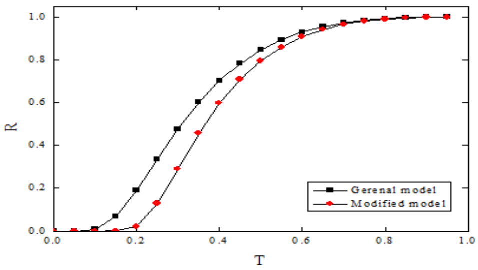

Figure 2: Relations between *R* and *T* for General Model and Modified Model

In the following part, we apply our model to the multilayer system, and the simulation results are presented. Figure 3(a) shows that connections between layers can substantially promote the fraction of propagation size, and the effect strengthens as the connection rate increases. In addition, we can see that the threshold decreases with the increase of connection rate. Figure 3(b) gives the simulation results of the threshold of the multilayer system. From the result, we summarize the following equation:

$$T_c = ae^{b\theta\varphi + c\theta\varphi^2}, \quad (4)$$

where $a = 0.188$, $b = -0.558$, $c = 0.158$ and $\varphi = 1 - \alpha$. It will allow us to predict the threshold of the multilayer system when connection rate and layer-switching cost ratio are determined. From Eq. (4), when $\theta = 0$ or $\alpha = 1$, the threshold value of the system reaches the maximum $T_c = 0.188$. That is to say, the system is equivalent to a single isolated network, so the information can not be propagated to the non-source layer.

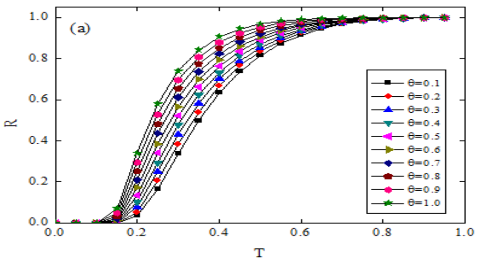

Figure 3(a): Relations between *R* and *T* for Different Connection Rate, $\alpha = 0.1$

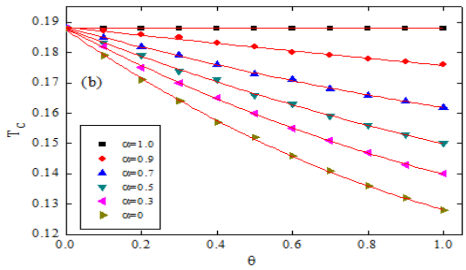

Figure 3(b): Relations between T_c and *θ* for Different Layer-Switching Cost Ratio

In scale-free networks, there are two types of special nodes: low-degree and high-degree nodes. Low-degree nodes refer to the nodes with a few neighbors and the number of this type of the whole network is large, just like the general users in real online social networks; High-degree nodes are very few nodes in the networks which have a large number of neighbors, as the so-called elite users.

In Figure 4, we show the impact on high-degree and low-degree nodes as the connection nodes on the multilayer networks. As we have mentioned above, the high-degree nodes are few in number in the system. Here, we pick out only a few nodes in each layer as the connection nodes. In Figure 4(a), it can be seen that the non-source layer with high-degree nodes as the connection nodes has better propagation characteristics, representing as lower threshold and larger propagation size. The results shown in Figure 4(b) that the type of connection nodes has little effect on the source layer. In our opinion, the connection nodes are the initial sources of information on the non-source layer. Therefor, the propagation results of non-source layer are more sensitive to the types of connection nodes.

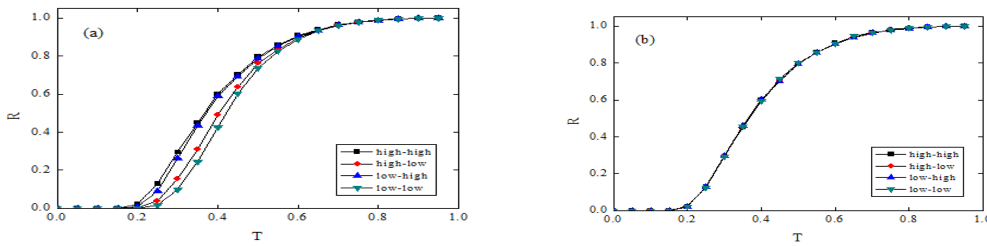

Figure 4: Relations between *R* and *T* for Different Type of Degree-Degree Connection between Source Layer and non-source Layer on the (a)Non-Source Layer, (b) Source Layer, $\alpha = 0.1$

Figure 5(a) and 5(b) show the simulation results in the cases of different parameter m in each layer respectively. Apparently, changing the structure of source layer has a greater impact on the propagation results. The variation on threshold value of case (I) is about 1.42 times of case (II). And the variation on propagation size of case (I) is over 2 times of case (II).

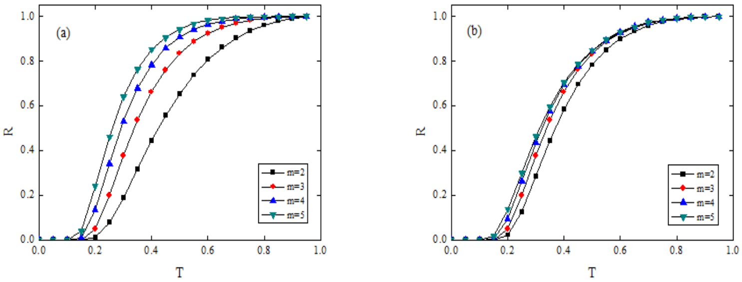

Figure 5: Relations between *R* and *T* for (a) Case (I): Different *m* of Source Layer, (b) Case (II): Different *m* of Non-Source Layer, $\theta = 0.2$, $\alpha = 0.2$

In Figure 6, simulation results of source layer and non-source layer in different cases are presented. It can be seen from the results that the change of structure of one network will change the propagation characteristics of this network and influence the other network. Remarkably, in Figure 6(b) and 6(d), we can find that for non-source layer, changing the parameter m of source layer is a more effective way to change the propagation results than changing the non-source layer's directly. The variation on propagation size of case (I) is at most 4 times of case (II). This illustrates that the structure of source layer plays a dominant role in the multilayer system.

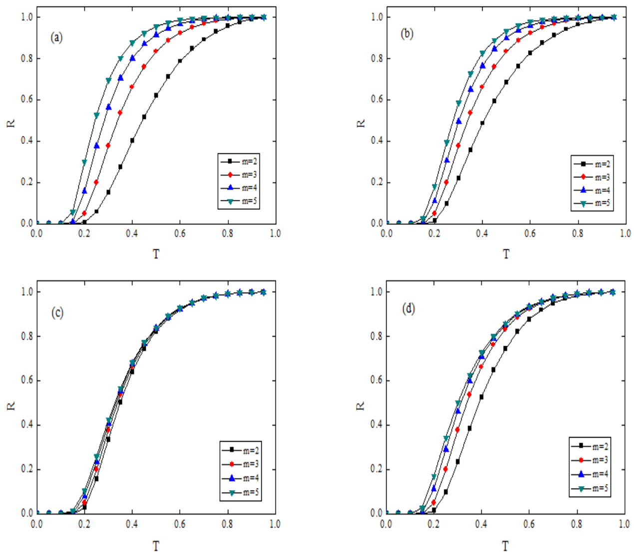

Figure 6: Relations between *R* and *T* for Case (I) On the (a) Source Layer, (b) Non-Source Layer; Relations between *R* and *T* for Case (II) On the (c) Source Layer, (d) Non-Source Layer, $\theta = 0.2$, $\alpha = 0.2$

CONCLUSIONS

In this paper, we have proposed a model in which the transmissibility is adjusted to the state of individuals and investigate information spreading on multilayer scale-free networks. It has been shown that connections between layers can substantially promote the fraction of propagation size, and the effect strengthens as the connection rate increases. In addition, considering the effect of layer-switching cost, the threshold equation is established so that we can predict the threshold of the multilayer system when connection rate and layer-switching cost ratio are determined. Furthermore, we have proved that the types of connection nodes can effectively affect the propagation results of the non-source layer but have little effect on the source layer. It has been shown that the structure of source layer plays a dominant role in the system of multilayer networks.

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